# Developing mathematical content knowledge for teaching: One pre-service teacher and her planning.

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This paper describes how a new pre-service teacher engaged with mathematical content in order to learn it for teaching, during practicum. The results show that the PST learned mathematical content by initiating and carrying out a preparation phase prior to planning. This phase involved searching through internet sites and making notes that were then used to support her lesson planning and teaching. Implications for pre-service teacher education are also presented.

## Introduction

In the field of mathematics education, there is renewed interest in *how* teachers plan for mathematics teaching, particularly regarding the decisions they make as they plan. Examples of these decisions include what key mathematics ideas, language and terminology, tasks, and pedagogical approaches to select for teaching. These decisions are important because they determine what is to be taught and how it is to be taught during lessons (Roche, Clarke, Clarke & Sullivan, 2014; Sullivan, Clarke & Clarke, 2012). Planning is a complex task for teachers, and even more challenging for pre-service teachers (PSTs) who are only beginning to learn about both mathematical curriculum and content for teaching, and as novice teachers have limited experiences to draw from to inform planning decisions (Ensor, 2001; Bailey, 2015).

Within initial teacher education programmes (ITE) professional practicum experiences provide the first real opportunities for PSTs to plan for mathematics teaching. Away from the university setting PSTs are required to take responsibility for planning, with varying degrees of support from associate (mentor) teachers, and university lecturers. They rely on curriculum and content knowledge gained from course work and mathematical content knowledge (MCK) they bring with them into the course and practicum setting (Ball, Thames, & Phelps, 2008). MCK is important knowledge for teaching, yet some PSTs have weak MCK and lack knowledge of the conceptual understandings of the mathematics they are required to teach, particularly at higher levels of the curriculum (Hurrell, 2013; Livy, Vale & Herbert, 2016). For some PSTs this knowledge can be developed while preparing to teach, which can occur as part of a planning process (Bailey, 2015; Roche et al., 2014).

The purposes of this paper are to describe how Ann (a pseudonym), a Year 1 PST in a three year ITE programme, developed MCK for teaching measurement (an area of mathematics she was unsure of), during her first practicum, and to suggest some implications of her experiences for both beginning PSTs and PST educators whose role it is to support PSTs to teach mathematics. These implications are cautiously made within the limitations of only one case that is reported in this paper. Ann's case is drawn from a larger longitudinal study which the author, a mathematics teacher educator, is presently carrying out. An aim of the larger study is to identify the issues PSTs face when preparing and teaching mathematics during practicum. Ann came to the author's attention during a debriefing session following

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her first practicum. During this session Ann presented several documents that she had written, in addition to her lesson plans, which she had created to help her learn the mathematical content she was required to teach. The author was curious about the content of these notes and how Ann used these to support her teaching. The specific research question then for this case study is, "how did Ann as a Year 1 PST engage with mathematical content needed for teaching three measurement lessons on practicum?"

## PSTs and planning

Learning to plan is recognised as an essential process for preparing for mathematics teaching and for developing teacher expertise (Mutton, Hagger & Burn, 2011). It is an important experience because it allows PSTs to make links between curriculum content, mathematical content and student learning (Koeppen, 1998). Planning lessons then are a common feature of ITE programmes (John, 2006), as was the case for Ann when participating in her Year 1 programme. Typically time is spent in courses learning what to write on various lesson planning templates, with the aim of providing PSTs with a detailed list of actions to follow when enacting planning while teaching (John, 2006; Roche et al., 2014). These actions provide structure and content for planning, but as Koeppen (1998) also found in her study of secondary PSTs, completed written plans served as "safety nets" (p. 405), for PSTs during teaching. One of her participants went to great lengths to research and record detailed information on his plan, which he referred to while teaching. This provided him with information as he taught, which made him feel more confident about what he was teaching, and increased his feelings of teacher identity.

While the completed plan is an important professional document, the actual process of planning also deserves attention, because it is a complex process demanding that PSTs become familiar with and make decisions about a range of curriculum resources, content, and practices, for the first time (Fernandez & Cannon, 2005; John, 2006; Mutton et al., 2011). For some PSTs this can be challenging as they "struggle to make sense of the cornucopia of decisions" available to them (John, 2006, p.498). It can be a time consuming process because unlike experienced teachers who draw on a repertoire of teaching knowledge, PSTs are only beginning to develop this knowledge and have minimal classroom teaching experiences (Borko & Putnam, 2000; Ensor, 2001). In their study of third year PSTs, Wilson and McChesney (2013), found that PSTs spent considerable time when planning for mathematics teaching, searching through a variety of resources, including the internet, looking for information to clarify curriculum content, including mathematical content.

## Knowledge for teaching mathematics

Content or subject knowledge is well recognised as an important category of knowledge needed for teaching (Grossman, 1990; Shulman, 1986). The well-known work of Ball and colleagues proposes two main domains of knowledge for mathematics teaching - subject matter knowledge and pedagogical content knowledge (Ball et al., 2008). Likewise, Anthony and Walshaw (2009) include both domains as important aspects of effective pedagogy for teaching mathematics. While there is much debate in the literature about the definitions and relationships between these two domains there is widespread agreement that teachers must know the subject matter that they teach, not just for themselves but also for implementation in the mathematics classroom. "Teachers who do not themselves know a subject well, are not likely to have the knowledge they need to help students learn the content" (Ball et al., 2008,

p.404). This is particularly relevant for some PSTs who may need to learn MCK in preparation for teaching. An additional challenge they face is to transform and use this knowledge for mathematics teaching (Fennema & Franke, 1992). Within an ITE course setting, PSTs have opportunities to do this within course work e.g. when preparing lesson plans for mathematics teaching, or when on practicum when planning for actual mathematics teaching. In their recent article about PSTs developing MCK during practicum, Livy et al., 2016, discuss the importance and value of practicum as a site for learning MCK, and note that the breadth and depth of experiences while on practicum, sustained engagement with MCK, and the quality of the PSTs learning experiences while on practicum are all important factors that can assist them to develop MCK.

## A pre-service teacher development model.

The theoretical framework for this study - *A revolving model of PST development* was a major finding from the author's master's thesis which focussed on how PSTs develop knowledge for mathematics teaching during their final year of an ITE programme (Wilson, 2010, 2012). The model is presented in Figure 1 below:

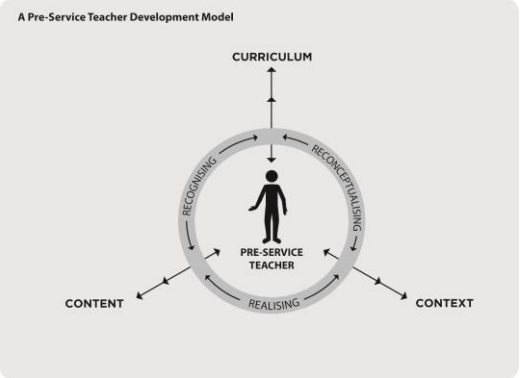



*Figure 1.* A revolving model of pre-service teacher development (Wilson, 2010, 2012).

It presents three domains of knowledge – content, curriculum and school contexts, along with three active phases of learning which describe *how* PSTs develop knowledge for teaching. These three phases are *recognising*, *reconceptualising* and *realising*. The first phase, *recognising* involves the PSTs noticing and identifying aspects of mathematics practices during course and practicum experiences. This phase draws on the work of both Mason (2002) and van Es and Sherin (2002) who describe noticing as an important process for developing knowledge for mathematics teaching, which for PSTs means identifying what is important or noteworthy for effective mathematics teaching and learning. The second phase, *reconceptualising* refers to the actions PSTs take to develop knowledge of content, curriculum and school contexts. In this phase PSTs act on what they have noticed and then transform this in some way for mathematics teaching (Ensor, 2001; Fennema & Franke, 1992). The third phase, *realising* refers to how PSTs realise or "make real" newly constructed knowledge for mathematics teaching. Within the constraints of an ITE programme this can be done by implementing knowledge in practice, when teaching

mathematics during practicum. This was the case for Ann who was required to teach three mathematics lessons during her first practicum.

In the model the PST is positioned at the centre as the novice professional who learns by recognising, reconceptualising and realising aspects of teaching. These phases of learning are presented in a circular image to emphasise the dynamic and non-linear nature of learning. The arrows depict the continual movement of learning for PSTs as they experience different aspects of their ITE course.

## Research design

For this qualitative study a case study design was selected because it is a bounded system which focuses on one case – Ann the year one PST, within the "real-world context" of the mathematics classroom, and on one aspect of her activity on practicum - her planning for mathematics teaching (Yin, 2014). The four week practicum occurred during the second semester in the last school term of the year, and after the completion of the ten week compulsory year one mathematics course. This course was not a course dedicated to mathematical content, however content knowledge was taught when it arose alongside pedagogical approaches. There were two assignments for the course, both of which focussed on planning for mathematics teaching, in the curriculum areas of measurement and geometry.

Ann was placed in an intermediate school (a school for year 7 and 8 students only) with thirty students in her Year 8 (12-13 year olds) class, and was given a "lower" ability group of approximately ten students to teach. The practicum assessment required her to plan and teach three consecutive lessons on the topic of measurement, in particular converting metric measures for length, weight and capacity, with tasks like 1.5 metres = 150 centimetres, 600ml = 0.6 of a litre. Each lesson was approximately 50 minutes in duration.

The author was Ann's visiting lecturer and consequently observed her teaching, and provided feedback on her planning and other documentation during and after practicum. During the debriefing session after practicum the author noticed the extra detailed work Ann had completed for mathematics teaching that was additional to her lesson planning documentation. It was at this point that Ann was invited to join the larger study, which included fifteen other year one PSTs. Ann provided written consent to be part of the study, and an individual interview was carried out with her once course sessions and assessment work had been completed. This was a consideration the author needed to adhere to as part of the ethical approval that was granted for the larger project.

Data were collected from three sources - the three mathematics lesson plans, Ann's extra resources which were pages of notes about measurement content, and the individual semi-structured forty five minute audio-taped interview that was carried out five weeks after the completion of the placement. The interview focussed on Ann's planning processes during practicum and included explanations of her lesson plans and the extra resources. These documents were available during the interview, which is a version of a stimulated recall process whereby documents are used to prompt and support discussions (Anthony, 1994). The interview was transcribed, provided to Ann for checking, and no changes were made. All documents, including the interview transcript were analysed using a process of content analysis, whereby entries and ideas written or described by Ann were identified, then collated into main themes. As a result of this process, the theme of how Ann engaged with and learnt the mathematical content for converting measures while planning, emerged. Results relating to this theme are now described, using the three phases of the PST development model.

## Developing mathematical content for teaching measurement

*Recognising.* Following a discussion with her classroom teacher she noted on her planning that the focus for her three lessons was "to convert between metric units using whole numbers and commonly used decimals". She recognised that this aligned with measurement achievement objectives from *The New Zealand Curriculum* (Ministry of Education, 2007) and recorded an appropriate achievement objective onto her three plans. She recognised that prior to planning her lessons she needed to learn this content, volunteering that "I just couldn't remember anything about measurement - I knew roughly there were 10mm in 1 cm…but I wasn't a hundred percent sure how to do it the quick way, like using times and divide". This "quick way" was required by the classroom teacher, and involved converting measures by multiplying, for example 5cm x 10 = 50 mm, and dividing, for example 800cm ÷ 100 = 8m.

*Reconceptualising:* Her first step to learn the content was to go directly to the internet where she did a general google search for converting measures. The search produced a New Zealand online mathematics resource that had been included in the course assessments, but she did not pursue these because this resource series was not used in her classroom setting. Two sites she did use were *Pinterest* and *YouTube.* On *YouTube* Ann typed in "teaching measurement converting" and then spent time selecting and watching videos that used the metric system of measurement. She described the process of searching as "sifting*"* through resources until she found something she deemed useful. She was drawn to information that was "easy to follow", particularly images which illustrated mathematical content in a chart or diagrammatical form, which she described as "visuals". As she watched she took "pages and pages of notes on refill", recording measurement concepts represented in different forms, such as symbols, text, diagrams and equations. Examples of her representations of length are reproduced in Figure 2.

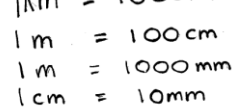
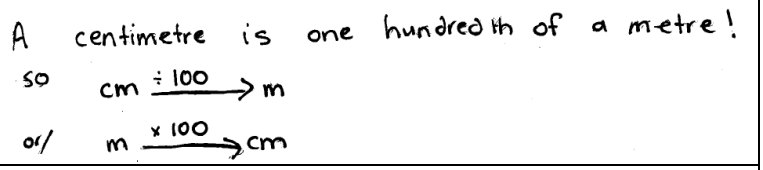
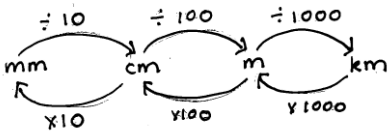
| | |
|---|---|
| Example 1: Symbols and conversions. |  |
| Example 2: Symbols, text, and a diagram for conversions. |  |
| Example 3: Diagram for converting length measures. |  |

*Figure 2* Examples of Ann's notes taken from internet sites.

Example 1 shows metric units for length and includes symbols e.g. km, m, cm, mm and equations showing relationships between these measures. Example 2 is a diagram using symbols and text, which Ann referred to as the "wordy way" to teach converting. The diagram in Example 3 shows how to convert measures using just symbols, and includes the

processes of multiplication and division, which was the quick way her classroom teacher had asked her to teach. Ann called this the "visual way".

She also made notes for weight and capacity which contained similar representations, but included measurement benchmarks, for example, a teaspoon can hold about five millilitres of water, and a loaf of bread weighs about 400 grams. On one page of notes she elaborated on these ideas by writing a script for teaching – "A paperclip weighs about 1 gram, hold one small paperclip in your hand. Does that weigh a lot? No! A gram is very light. That's why you will often see things measured in hundreds of grams". As she wrote the notes she was not only learning the mathematics for herself, but also thinking about how it could be used when teaching.

Although she only taught two lessons about length and one about weight, she also wrote notes related to volume and capacity. She did this just in case she was asked to do some additional teaching, and so that she had information for future use. "I thought I might as well put all the effort in so I can refer to them and go back to it". These notes were detailed and included definitions of mathematical terms such as "volume is the space a solid, liquid or a gas takes up", writing a formula for measuring the volume of a rectangular prism, and drawing a diagram of a cube with each dimension labelled. She also recorded facts like "whilst some objects may have the same volume, the weight/mass may differ" followed by examples which included - "1L of air is not heavy, 1L of lead is very heavy, and 1L of water is moderately heavy". These are further examples of learning the mathematics herself while also thinking about how the content could be used in a lesson.

*Realising:* Having written twelve pages of notes from her internet search, the next phase in this learning cycle was for Ann to "make real" this measurement knowledge for use in the classroom, by planning her three lessons. She adhered to the classroom mathematics lesson format and inserted examples from her notes as part of her lesson, in particular the "wordy way" and the "visual way", examples 2 and 3 from Figure 2, in the teacher actions section on her plan. She did this intending to use the examples to teach the students how to convert the measures, and to have information readily available while teaching, to remind her of the content she was teaching. The last indicator of Ann's engagement with the mathematics content occurred when she was actually teaching the lessons. Ann explained that as she taught she relied less and less on the notes recorded in her planning. While she taught she was able to do the problems mentally and by lesson three no longer needed to refer to the mathematical content recorded on her plans, saying "in the end I could just do it!"

In summary the process involved Ann *recognising* the need to learn mathematical content while planning and prior to teaching, using the internet as the main source of information, *reconceptualising* this by writing resources to support her planning, then *realising* the content by teaching the lessons.

## Discussion and Conclusion

Ann's experiences emphasise the importance of MCK as necessary knowledge for mathematics teaching (Ball et al., 2008), and highlights that for PSTs who are uncertain about the mathematics they need to teach, the act of planning provides an opportunity to learn content prior to teaching (Roche et al, 2014; Livy et al., 2016). The process Ann initiated involved her searching the internet, choosing familiar sites, and then selecting various representations of the mathematical content she was required to teach. This was a time consuming process as she navigated, noticed and selected content from these sites (Wilson and McChesney, 2013). She needed to do more than just read the content on the

internet, recognising that she had to engage with it in some way, in order to learn it prior to teaching. Her active decision to make substantial notes which included text, symbols, diagrams, definitions and mini scripts for teaching, enabled her to make sense of the content prior to planning, and the notes also became a valuable resource which became part of her teaching repertoire for future use.

For Ann, initiating and taking the extra step of learning mathematical content by searching and note making prior to planning, was a necessary step for planning. Fernandez and Cannon (2005) suggest that planning can be broken into two stages – preparation and planning. For PSTs like Ann who need to learn mathematical content adding in a preparation phase, prior to teaching would provide them with an opportunity to identify and learn mathematical content for their lessons.

Having prepared the content Ann was then able to begin planning, which allowed her to engage with the mathematics again by including key ideas on her plan. She did this by writing the "visual" way and the "wordy" way from her notes, into each lesson sequence. Roche et al., (2014) acknowledge key mathematics ideas as an important component of planning. As she taught, she was able to refer to these representations, and they acted as prompts to remind her of the content as she taught (Koeppen, 1998). Her final opportunity to engage with the mathematics content occurred when teaching the lessons. By repeatedly teaching the mathematical content over the course of the three lessons, Ann was able to let go of the safety net provided by her plans, and commit the content to memory.

During mathematics education courses prior to their first placement PSTs would benefit from course experiences where they are supported by peers and PST educators, which require them to prepare mathematical content prior to planning. This would include identifying from the curriculum what they need to teach, sourcing reliable resources containing representations and pedagogical practices to align with this, and then engaging with the content in some way (Bailey, 2015). Ann's process of making notes worked for her, another suggestion is for PSTs to make notes of mathematical content they notice and encounter during course sessions. Given the vast quantity of information on the internet, and the ease at which PSTs access this information, there would be value in PST educators recommending appropriate sites, and providing guiding principles for the critique and selection of content from these. PSTs would also benefit from practising writing detailed lesson plans, particularly at the higher levels of the curriculum, as part of course work where they could be supported by peers and PST educators to identify and learn the mathematics that needs to be taught (Roche et al., 2014, Sullivan et al., 2012). Finally, PST educators need to continue to value and prioritise practicum experiences as opportunities for PSTs to not only teach mathematics, but also to learn mathematical content for themselves (Livy et al., 2016).

This paper has focussed on one PST, and there are major limitations when suggesting implications from one case, however this in-depth analysis of Ann's actions to learn mathematical content while planning during practicum contributes to the on-going focus and direction of the larger study. The author is continuing to investigate the planning and teaching experiences of the PSTs in this study, and as a consequence has designed and implemented course work books with dedicated space for PSTs to record and engage with mathematical content during course sessions, planning formats which require PSTs to record and emphasise mathematical content, and workshop tasks where they practise preparing and planning content for teaching.

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